Game Theory

Social Intelligence

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Reminders: Dominance

Def. (strong dominance): a strategy s for a player p *strongly dominates* s' if the outcome is better for *every* choice o strategy for other players.

Def. (weak dominance): a strategy *weakly dominates* if it is better on (at least) one strategy of other players and n worse on any other.

Def.: A *dominant strategy* dominates all others.

Reminders: Pareto optimality/dominance

Def. (Pareto optimality): an outcome is *Pareto optimal* if no other outcome would be preferred by all the players.

Def. (Pareto dominance): an outcome is *Pareto dominated* if all players would prefer some other outcome.

Dominance in Prisoner's Dilemma

Note: both Alice and Bob have a dominant strategy, i.e. we have a dominant strategy *equilibrium*

Def. (Nash equilibrium): a selection of strategies for each player such that no player can benefit by switching his/her strategy if all other players' strategies are unchanged.

Remark: the *dilemma* in the prisoner's dilemma is due to the fact that the Nash equilibrium of both prisoners cooperating is Pareto dominated by $(-1, -1)$.

Note: a Nash equilibrium can arise even without the existence of a dominant strategy.

Comments on Game Theory

Remark: if

- the prisoner's dilemma game is being iterated
- the players are allowed to have memories and identify their opponent

this can lead to solutions which avoid the equilibrium.

Note: Tit-For-Tat and very related strategies prove to be remarkably stable and robust solutions.

Remark: if one has a Pareto-optimal point which is also a Nash equilibrium, then we call that a *solution* of the game.

Zero-Sum Games

Consider: zero-sum games. Then we need to consider onl the payoff P for one of the players, the other will follow as $-P$.

2-Finger Morra: remember the payoff matrix:

$E \backslash O$	1	2
1	$2 \backslash -2$	$-3 \backslash 3$
2	$-3 \backslash 3$	$4 \backslash -4$

Goal: find *solution*

Zero-Sum Games: Solution

Scenario 1: force E to begin, O to follow. This is an advantage for O. It is easy to see that E is guaranteed an outcome of $U_E \geq -3$.

Scenario 2: force O to begin, E to follow. O can ensure an outcome with $U_E \leq 2$.

Mixed Strategy

Note: revealing a strategy gives the second player an advantage. For, if second player plays $[p : 1; (1-p) : 2]$, the expected utility is

$$pU_{O=1} + (1-p)U_{O=2}$$

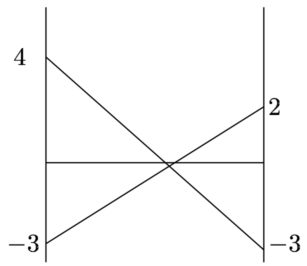
If $U_{O=1}$ and $U_{O=2}$ are different, pick the best as *pure* strategy.

Utilities for Mixed Strategies I

Assume: E moves first, without O knowing the move, but knowing p in the strategy $[p : 1; (1-p) : 2]$ of E. Then if

1. O chooses 1, then $\mathbf{E}(U) = 2p - 3(1-p) = 5p - 3$
2. O chooses 2, then $\mathbf{E}(U) = -3p + 4(1-p) = 4 - 7p$

O will always pick the minimum of both. E will pick p such that is maximal, $U = -\frac{1}{12}$.



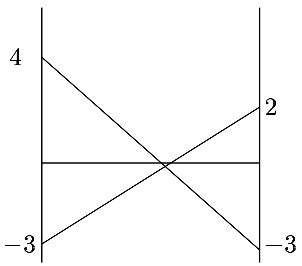
Utilities for Mixed Strategies II

Assume: O moves first, probabilites $[q : 1; (1-q) : 2]$. If

1. E picks 1, then $\mathbf{E}(U) = 2q - 3(1-q) = 5q - 3$
2. E picks 2, then $\mathbf{E}(U) = -3q + 4(1-q) = 4 - 7q$

E picks the maximum of both. O will pick q such that is minimal, $U = -\frac{1}{12}$.

Note: The two U values enclose the true value, which is therefore $U = -\frac{1}{12}$. It turns out that $p = \frac{7}{12} = q$.



Minimax Equilibria

Bottom Line: there exists an *equilibrium*, a *minimax* equilibrium which is Nash equilibrium.

von Neumann: every two-player zero-sum game has a minimax equilibrium on mixed strategies. Also, in zero-sum games, Nash equilibria are minimax equilibria.